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THE CHARACTERIZATION OF ACOUSTIC VARIABILITY

IN THE OCEAN

by

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ABSTRACT:

The temporal and spacial variation in one-way transmission loss as experienced in the ocean due to short term temporal and small scale spacial variation in the acoustic environment is examined. This variation is characterized as a function of the transmission frequency, transmission range, source and receiver depths, predominant thermal structure and geographical locality. The results obtained clearly indicate that variability in transmission loss is indeed dependent upon the relative position of source and receiver within the acoustic medium as well as the nature of the acoustic medium. Suggestions are made as to the nature of the influences which control this variation.

Prepared by:

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Executive Summary

Preliminary results of the presently reported study are contained in a Naval Postgraduate School Report NPS55ZA081A titled "The Characterization of Variability in Transmission Loss in the Ocean," August 1971 by the author. Since that date the data base has been expanded to 10,000 data points. In addition, at the suggestion of the results of preliminary analysis the data has been parameterized with respect to the nature of the transmission path as transmission loss variability appears to be a function of this path. The assumption of the equivalence of transmission loss in both directions along a transmission path is reconsidered and the number of analysis techniques applied has been increased as has the range of oceanographic conditions under which transmission loss is investigated.

Specifically, this study investigates the relationship of acoustic variability in the ocean to selected characteristics of the local acoustic environment and to the location of the acoustic source and receiver within this environment. The data used are from the AMOS project consisting of in-situ measurements of transmission loss at frequencies of 2.2, 8, 16, and 25 KC. Transmission ranges are from 800 to 30,000 yards and source and receiver depths are from 12 to 500 feet. As a result the conditions under which acoustic variability is investigated is within the range of conditions experienced by surface and subsurface ship mounted sonar.

The specific results are concerned with the time dependent nature of acoustic variability and the effect on the magnitude of acoustic variability by parameters such as sea state, transmission depth and range and temperature gradient. Also, the magnitude of acoustic variability typical to selected regions in the North Atlantic as a function of time of year are presented.

In addition to determining which environmental and system parameters affect acoustic variability and the nature and magnitude of these effects, inferences are drawn with respect to the physical processes which lead to the observed nature of acoustic variations.

I. Introduction.

The objective of this study is to measure in-situ the variability of sound transmission loss in the ocean from an omnidirectional source and to determine the relationship of this variability to selected parameters describing the acoustic environment and the physical location of the acoustic hardware.* Experience has shown that when transmitting from a point S (source) with an acoustic energy E_S that the amount of acoustic energy E_R received at a point R (receiver) varies with time. Due to energy absorption during transmission and to spreading losses $E_R < E_S$. The quantity $10 \log(E_S/E_R)$ is defined as transmission loss and shall be denoted by TL. For any pair of points (S,R) in the ocean TL is a random variable. This study considers how the statistical properties of this random variable are related to the nature of the acoustic environment and to the relative positions of the points S and R.

The temporal variation in TL is a result of the particular nature of the ocean as an acoustic medium. The ocean has a density structure which experiences considerable variation with depth, but the variation of salinity and an increasing static pressure with depth also contribute to this density variation.

Transmission of energy through any medium of varying density will result in a defraction of that energy from a straight path. Energy will also be reflected at the boundary of the acoustic medium.

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*This study is a continuation of a study by the present author first reported in Naval Postgraduate School Report NPS 55ZA71081A titled The Characterization of Variability in Transmission Loss in the Ocean.

Thus, the amount of energy received at point R from point S depends upon how much of the original energy transmitted is focused on R due to refraction and reflection processes. As the density structure in the ocean experiences variation as a result of ever present turbulence and the action of internal wave motion [3] and as the nature of the reflecting surfaces change, the amount of energy focused on point R will vary with time.

If points S and R are maintained at the same depths and range separation but translated horizontally within the local acoustic medium additional variation in TL will be experienced. This is a result of the spacial variation of the density structure of the ocean and of the spacial variation in boundary reflection and absorption characteristics. Recent studies [2,5,6] have shown that the ocean temperature profile is largely comprised of layers of "quasihomogeneous" water a few meters thick separated by very high gradient sheets a few centimeters in thickness. This suggests a temperature structure consisting of lenses of isothermal water sandwiched between similar lenses of water at slightly different temperatures. Moreover, these lenses do not necessarily lay perfectly horizontal and can vary in thickness over their extent. Thus, a horizontal translation of points S and R in such an environment will lead to experiencing additional density profile variation.

Both temporal and spacial sources of variation are important in operational acoustic systems. In a fixed passive acoustic system

the limiting range and consequently system capabilities will vary with time. In an acoustic system employing aircraft planted sonabuoys the array pattern is selected on predicted mean transmission characteristics. However, due to the simultaneous presence of both temporal and spacial variation in TL, the proposed array pattern is often not optimum and knowledge of expected acoustic variability would be useful in selecting a more optimum pattern.

To measure temporal variation alone for a given transmission range and source and receiver depths, a series of equally time spaced measurement of TL between fixed points S and R are required. To measure spacial variation alone, a given transmission range and source and receiver depths pairs of points, S and R, must be appropriately spaced throughout the local acoustic environment and observations of TL between these pairs of points be observed simultaneously or nearly so. To simultaneously measure temporal and spacial variation, the measurements of TL between equally spaced pairs of points S and R placed throughout the local area must be taken at different times and comparisons of the observed TL made as a function of time between the respective observations.

The relative magnitudes of temporal and spacial related variations in TL are unknown. However, in the absence of boundary related spacial variability in TL one can postulate that, given sufficient time, temporal variability between two local points will equal spacial variability within that local region at any given time. The rationale behind this postulate is that the dynamic forces of

turbulence, currents and internal wave motion which cause spacial variability are the same forces which cause temporal changes in density structure in the region directly between points S and R. Therefore, given sufficient time these forces can create temporal density structure variation of the same magnitude as experienced spacially. An interesting question is whether this time is of the order of minutes or hours or days or months. It should also be noted that the total amount of TL variation observed if one experiences both temporal and spacial variation simultaneously cannot exceed the variability of the largest of the two sources alone.

This study considers the statistical properties of total variability (both temporal and spacial) in TL observed within a localized area with the consideration of temporal variability experienced in time periods of less than 90 minutes. The data utilized is of such a nature to measure both temporal and spacial variation simultaneously. In some cases temporal variation will be separable from spacial variation and these cases will be discussed in the section on results.

The following is the model upon which the ensuing analysis is based.

II. A Model of the Acoustic Environment.

If one considers small enough ocean regions and short enough periods of time the statistical properties of the random variable TL will not change significantly. The maximum spacial and temporal

dimensions of such regions will depend upon the relative stability of the major density profile characteristics, the turbulence structure and the boundary characteristics over such regions and times. It will be assumed that in the absence of shallow water and sharp water mass boundaries that regions of less than 15 nautical miles in diameter, for times less than 90 minutes, will be statistically homogeneous with respect to TL. For such times these regions will be defined as acoustically homogeneous in that the statistical properties of the acoustic transmission loss are independent of time and position as long as transmission frequency and range and the source and receiver depths are maintained.

On this basis TL between fixed points, S and R, within an acoustically homogeneous region may be modeled as a stationary stochastic process whose mean is a function of the mean features of density profile and whose autocovariance function is a function of the time dependent density irregularities superimposed upon the large scale density features. Denote this process, where $X(t)$ describes the TL in db at time t, by $X(t);\ t \geq 0$. It is convenient to characterize this process by its mean

$$\mu = E(X(t)) \tag{1}$$

and by its autocovariance function

$$C(\Delta t) = E(X(t)-\mu)(X(t+\Delta t)-\mu). \tag{2}$$

It is the acoustic variability as described by this autocovariance function with which we are principally concerned.

Let us examine the nature of the autocovariance function $C(\Delta t)$ of the process $X(t)$ with respect to the nature of the forces contributing to the variability in TL. It can be expected that $C(\Delta t)$ will generally decrease with increasing Δt due to the action of turbulence on the density microstructure of the acoustic medium [7]. Superimposed on this generally decreasing function may be periodicities due to the cyclic effect on TL as a result of surface and internal wave motion [3,7]. In some cases, the presence of internal waves might be the dominant force behind TL variability. In such situations, the autocovariance function would be a periodic function being gradually eroded by turbulence effects until $C(\Delta t) \to 0$. A hypothetical representation of this is presented in Figure 1.

Two particularly interesting parameters characterizing variability may be obtained from the function $C(\Delta t)$. The first parameter is $C(0)$ and as can be seen from equation (2) is equal to $Var(x(t))$, the total variability in TL one would expect to experience in a given environmental situation. The second parameter is that value of Δt for which the function $C(\Delta t)$ becomes essentially zero thereafter. This value, denoted by Δt^*, then gives the minimum time interval between stochastically independent observations of TL.

These two parameters, $C(0)$ and Δt^*, may be estimated for a given stationary process if that process is observed either continually

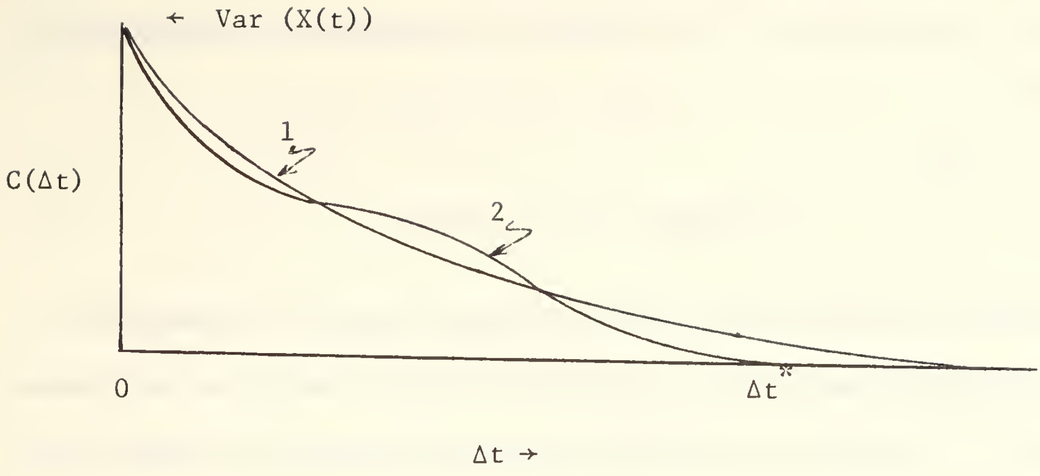

Figure 1. Autocovariance Function of Transmission Loss.

Curve 1. A hypothetical autocovariance function of TL under the action of turbulence.

Curve 2. A hypothetical autocovariance function of TL under the action of both turbulence and internal wave motion.

or at discrete increments of time. Then by observing such processes under varying acoustic conditions it should be possible to determine how $C(0)$ and Δt^* vary as a function of the acoustic environment.

However, if one assumed the process $X(t)$, $t > 0$ is ergodic, it is not necessary to observe the entire process to determine $C(0)$ and Δt^*. Specifically, if the process of transmission loss is observed at two times separated by Δt and such pairs of transmission losses are observed many times at varying values of Δt for the same process, or equivalently from processes with the same autocovariance functions, then this information may be used to estimate $C(\Delta t)$ and

hence, $C(0)$ and Δt^*. This may be accomplished in the following way:

Let

$$Y(\Delta t) = X(t) - X(t+\Delta t) \tag{3}$$

represent the difference between two measurements in transmission loss observed under the same general acoustic conditions and consequently having the same expected transmission loss but separated in time by Δt.

Then for a given $\Delta t \geq 0$

$$E(Y(\Delta t)) = 0 \tag{4}$$

and
$$\begin{aligned} \mathrm{Var}(Y(\Delta t)) &= E(Y(\Delta t)^2) \\ &= \mathrm{Var}(X(t)) + \mathrm{Var}(X(t+\Delta t)) - 2C(\Delta t) \\ &= 2(C(0) - C(\Delta t)) \end{aligned} \tag{5}$$

The value of $\mathrm{Var}(Y(\Delta t))$ may then be determined as a function of Δt from the observations of $Y(\Delta t)$ corresponding to all pairs of observations of transmission loss from processes with the same autocovariance functions, $C(\Delta t)$. $C(0)$ may then be represented by

$$C(0) = \mathrm{Var}(Y(\Delta t^*))/2 \tag{6}$$

where $\mathrm{Var}(Y(\Delta t^*))$ is the value of $\mathrm{Var}(Y(\Delta t))$ when $C(\Delta t)$ vanishes or when $\mathrm{Var}(Y(\Delta t))$ no longer increases with Δt. Then Δt^* is the time interval where this first occurs.

Thus the autocovariance function may be expressed as

$$C(\Delta t) = C(0) - \frac{1}{2}\,\text{Var}\ Y(\Delta t)$$

$$= \frac{1}{2}\,\text{Var}\ Y(\Delta t^{*}) - \frac{1}{2}\,\text{Var}\ Y(\Delta t). \qquad (7)$$

If several pairs of observations of TL cannot be obtained from the same process (as is the case with the presently available data), then it is necessary to pool information derived from different processes with the same autocovariance function. The problem herein is that of identifying the situations under which these common autocovariance functions occur. The approach taken in this study was to label each pair of TL observations with respect to selected environmental and system parameters which could potentially affect the nature of the autocovariance function. Then by comparing the magnitude and temporal nature of the function $\text{Var}(Y(\Delta t))$ and hence of $C(\Delta t)$ for various values of these parameters and combinations thereof, conditions which led to different autocovariance functions were detected. In this manner, the various parameters which affect acoustic variability were noted and the nature and magnitude of this variability was determined for the various values of these parameters.

III. Nature of the Data.

The data utilized in this study was provided by the U. S. Underwater Sound Laboratory, New London, Connecticut, and is derived from the raw data collected during the Acoustic, Meteorological and

Oceanographic Survey (AMOS) conducted from June 1949 through April 1953 [4]. There were nine cruises staged during these four years which covered the North Atlantic, the Norwegian Sea, and to a lesser extent the Mediterranean Sea.

Two ships were employed during each cruise. One acted as a transmitting platform and the other as a receiving platform. During each of the cruises a series of widely separated stations were occupied. These stations served as the focal point for the data measurement and collection. Within the local area of each station acoustic data was collected at several transmission ranges between 800 and 30,000 yards. The acoustic energy was generated using a broad band, omnidirectional source and the receivers were tuned to receive acoustic energy at 4 frequencies; 2.2 KC, 8 KC, 16 KC, 25 KC. At each transmission range the transmission loss measurements were observed for the four frequencies at various source/receiver depth combinations. The source and receiver depths varied from 12 to 500 feet. The specific data recorded was transmission loss in decibels (db), as a function of the source/receiver depths and range between ships. Also recorded was appropriate oceanic and meteorological environmental data. Table 1A of the appendix lists the information that was recorded.

Accuracy of the data becomes particularly important in considering the accuracy of any results obtained through the use of that data. As the author was not involved with the original AMOS experiment no concrete

information is available on these accuracies. However, the accuracy with which the data was recorded provides an upper limit on the accuracy of the data. These accuracies are provided in Table 2A of the appendix.

As indicated previously only a single pair of measurements of TL are available within an acoustically homogeneous region at any given range and source/receiver depth combination. However, at most ranges several source/receiver depth combinations are experienced. The source/receiver configuration for the paired observations is as follows:

1) The first observation of a pair measured the TL of a signal generated at depth d_1 and received at depth d_2 and the second observation of that pair measured the TL of a signal generated at depth d_2 and received at d_1.
2) The transmission ranges of the two paired observations were not always identical but it was always the case that the maximum difference in transmission range was 500 yards.

Figure 2 depicts the geometric nature of the available data.

These data characteristics had the following implications with respect to analysis and interpretations.

The first implication is that a spacial contribution of variability is present in the data due to the two paired transmissions originating and ending at different locations. The magnitude of this

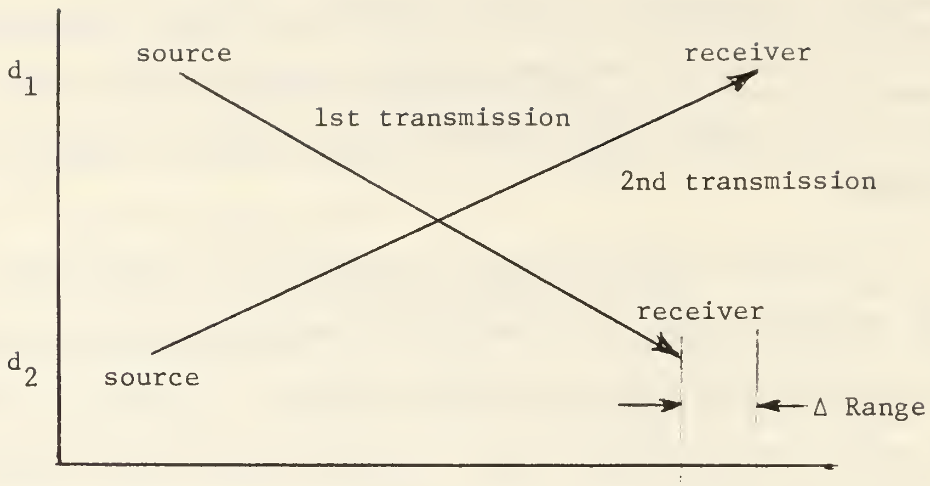

Figure 2.

Available pairs of transmission paths for a given range and source/receiver depth combination.

spacial variation, however, may be less than that of the overall spacial variability characteristic to the local region because both transmissions in question are confined to the region directly between the two ships. If this is the case TL variability will vary with time between transmissions to reflect temporal changes in density structure. Consequently, in spite of the presence of a spacial component of variability some indication of the nature of temporal variation may be available.

The second implication is that the expected difference in transmission loss between the "downward" and the "upward" transmission which were paired may not be zero as required by equation 4 even though the endpoints of the transmissions are maintained at the same

depths. This is a consequence of the negation of the applicability of the acoustical reciprocity theorem [8] dealing with the transmission of acoustical energy in a medium of uniform density by the presence of the stratified density structure of the ocean related to variation in temperature, pressure and salinity as well as the temporal variation in density. As it is not known how or to what extent these density variations in the acoustic medium will affect the mean directional difference in transmission loss statistical methods were implemented to estimate this mean directional difference as a function of environmental factors and source and receiver relative position. The results of this analysis is presented in a later section.

The second data characteristic of range differences between the two observations in a pair was treated in the following manner. The observed change in transmission loss was adjusted for the effect of this range differential on the basis of a piece-wise linear approximation of transmission loss vs. horizontal transmission range. This linear approximation seemed reasonable in the interest of keeping the range adjustment computation simple and yet maintaining sufficient accuracy. Plotting transmission loss vs. transmission range for each source/receiver depth combination at each station revealed that the resulting relationships exhibited the same general shape and that these relationships could be approximated by two linear segments with the break point at 5,000 yards. This slope change occurs roughly at the range of transition between spherical and cylindrical spreading.

As the slopes of these two linear segments are a function of the local environmental conditions, and thus varied from station to station, it was necessary to estimate the slope for each segment at each station for each source/receiver depth combination. This was done on the basis of the transmission loss observed at the various ranges occupied on a given station. Then the change in transmission loss observed at a particular station and source/receiver depth was adjusted using the corresponding slope of TL vs. transmission range in the following manner.

$$\Delta TL_{(ra)} = (TL_1 - TL_2) - b x (\Delta \text{ range}) \qquad (8)$$

where $\Delta TL_{(ra)}$ represents the range adjusted observed change in TL, b is the slope of the corresponding linear segment and Δ range is the range differential within a pair of observations.

Admittedly this adjustment procedure is subject to error; in particular, the error incurred in using a linear approximation and the error incurred in estimating the slope of this relationship between TL and transmission range.

It was felt, however, that in general the total error inherent in this procedure could be restricted to at most $\pm .5$ db by making no adjustments with range differentials within pairs of observations of more than 500 yards. Errors of such magnitude were considered acceptable in light of the original accuracy in measuring transmission loss in the AMOS experiment. In actual practice, this error was

further limited by the fact that few adjustments in change in transmission loss were made for range differentials greater than 250 yards.

In addition, an analysis was performed to see if the presence of the range differential and the resulting adjustment procedures contributed significantly to the observed variability in TL. The nature and results of this analysis is discussed in Sections V and VI respectively.

IV. Data Reduction.

The original data collected during the AMOS experiment consisted of the following. At each station occupied (a total of 163 stations on the 9 cruises) a transmitting ship positioned itself at the center of the station. Acoustic energy was generated at selected depths by suspending a broad banded omnidirectional source on a cable beneath the ship. Transmitting depths ranged from 12 to 500 feet. At distances varying from 800 to 30,000 yards a second ship suspended four receivers tuned to receive acoustic energy at the frequencies of 2.2 KC, 8 KC, 16 KC and 25 KC. Reception depths also ranged from 15 to 500 yards. By systematically lowering and raising the transmitter and receiver transmission was experienced between a variety of source/receiver depth combinations at each range investigated. Typical values of source and receiver depths are 20, 50, 100, 200 and 450 feet.

Figure 3 represents a typical configuration of the acoustic system and the chronological order in which the source and receiving

depths were occupied. In Figure 3 t_i represents the $i^{\underline{th}}$ time in the time sequence at which a signal was transmitted and received.

Referring to Figure 3 it is seen that at time t_6 the source is at a depth of 50 feet and the receiver is at a depth of 450 feet. At time t_{22} the depths of the source and receiver are reversed. It is under these geometric conditions that two observed transmissions were selected for possible pairing.

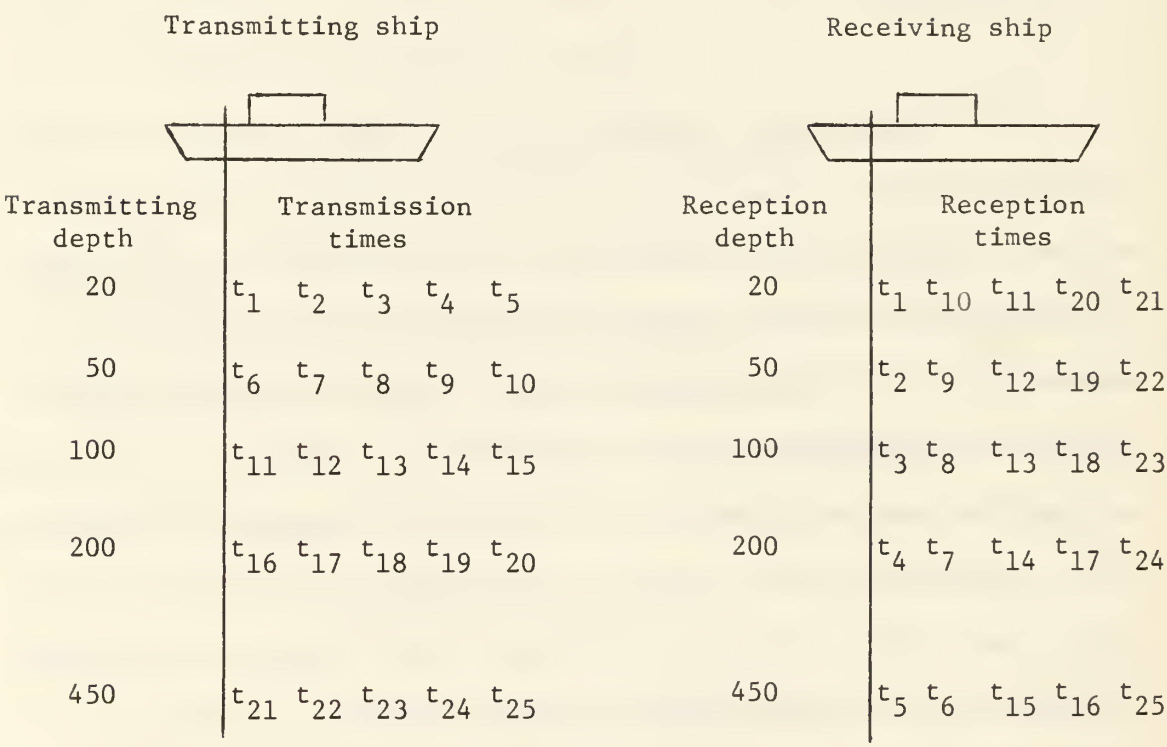

Figure 3.

Time-Depth configuration of typical acoustic system during AMOS experiment.

Additional conditions were required prior to pairing. These conditions were:

1) Both transmissions must be observed at the same station.

2) Both transmissions must be at the same frequency.

3) A range difference of less than 500 yards was required.

4) A time difference between transmission of less than 90 minutes was required.

All transmissions not forming pairs under these conditions were discarded. This process resulted in nearly 10,000 pairs of measured TL covering a wide range of conditions.

For each pair of observations the difference in TL (denoted ΔTL) was calculated, corrected for any range differential using equation (8) and recorded along with pertinent environmental and system parameters such as transmission frequency and range, source and receiver depths, sea state and water temperature at various depths. The entire list of data recorded per transmission pair is given in Table 1A of the appendix. This collection of range adjusted ΔTL's and the corresponding data describing the conditions under which TL was observed formed the data base for the ensuing analysis.

As preliminary analysis suggested that the nature and magnitude of the variability in TL may depend upon the nature of the path followed by the acoustic energy in traveling from point S to point R, each pair of transmission was evaluated as to the most likely transmission path experienced.

This was accomplished using equations derived as a product of the original analysis of the AMOS data by Marsh and Schulkin [4]. These equations are based on the refraction characteristics of the acoustic medium assuming typical temperature profiles, on observation confirmed theoretical relationships of TL to physical phenomenon and upon the relative positions of the source and receiver.

Six path types were considered and are designated transmission modes 1-6 as follows: (see Figure 4)

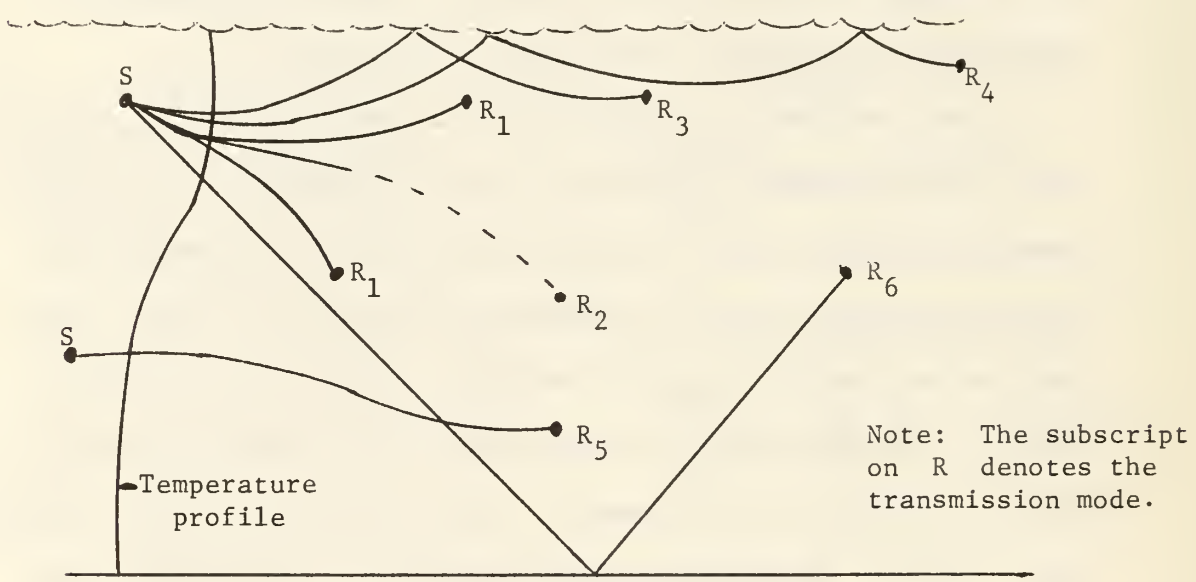

Figure 4.

Transmission paths of acoustic energy as a function of source and receiver positions.

Mode 1. Direct transmission between S and R with S and/or R in the mixed layer. No energy contact with the surface or bottom.

Mode 2. Transmission across or below the layer depth by defraction of the energy in the limiting ray.

Mode 3. Transmission experiencing one reflection off the surface.

Mode 4. Transmission experiencing at least two reflections off the surface.

Mode 5. Direct transmission entirely below the mixed layer.

Mode 6. Transmission experiencing a bottom reflection.

V. Analysis Techniques.

Two different types of analyses were performed. These were concerned with the expected difference in "downward" TL minus "upward" TL and the variance in TL as a function of selected environmental and system parameters.

a) Analysis of Mean Directional Difference.

The development in section II with respect to using the differences in two measurements of TL taken under the same geometric configuration within an acoustically homogeneous environment to estimate variability in TL required that the expected TL be the same for both transmissions. Preliminary analysis showed that this was not necessarily the case when the two transmissions "crossed" rather than were "parallel." Moreover, it was apparent that the mean difference due to the directional factor (upward transmission vs. downward transmission) depended upon the nature of the acoustic environment and the relative positions of the source and receiver.

If the mean directional difference in TL was known as a function of the conditions under which each pair of TL measurements was observed then equations (3)-(5) of section II could be revised appropriately as follows:

$$Y_{adj}(\Delta T) = (X_d(t)-\mu_d) - (X_\mu(t+\Delta t)-\mu_\mu)$$

$$= Y(\Delta t) - (\mu_d-\mu_\mu)$$

$$= Y(\Delta t) - \mu_\Delta \tag{9}$$

where the subscripts d, μ, and Δ represent respectively downward, upward and downward minus upward transmission. It follows that

$$E(Y_{adj}(\Delta t)) = 0 \tag{10}$$

and

$$\mathrm{Var}(Y_{adj}(\Delta t)) = E(Y_{adj}(\Delta t))^2$$

$$= \mathrm{Var}(X(t)) + \mathrm{Var}(X(t+\Delta t)) - 2\ \mathrm{cov}(\Delta t)$$

$$= 2(C(0) - C(\Delta t)). \tag{11}$$

This mean directional difference in TL, μ_Δ, was estimated as a function of selected acoustic and system parameters using multiple regression techniques. Each of the four frequencies was treated individually. The dependent variable in the regression was $Y(\Delta t)$. The independent variables are listed in Table 1.

Table 1.

Independent Variables in Multiple Regression for

Directional Difference in TL Analysis

- Range (KY).
- Sea state (4 levels).
- Transmission mode or path type (6 levels).
- Vertical position of the source and receiver with respect to the layer depth (3 levels).
- Δ depth between source and receiver (feet).
- Δ temperature between source and receiver (°F).
- Average temperature gradient between source and receiver (°F/ft).

The variables sea state, mode and relative positions of the source and receiver were entered as qualitative variables. This was accomplished for sea state by creating four independent variables X_1, X_2, X_3, X_4 representing respectively the presence or absence of sea state 1, 2, 3, ≥ 4. If the $i^{\underline{th}}$ sea state occurred then $X_i = 1$ and the remaining $X_j = 0 \quad j \neq i$.

Transmission mode was handled in the same manner using six variables $X_5 \ldots X_{10}$. To describe the selective position of the source and receiver with respect to the mixed layer depth three qualitative variables X_{11}, X_{12}, X_{13} were defined. $X_{11} = 1$ if both the source and receiver are in the mixed layer and zero otherwise. $X_{13} = 1$ if both the source and receiver are below the mixed layer and

zero otherwise. $X_{12} = 1$ if one end of a transmission is within the mixed layer and one end is below the mixed layer and zero otherwise. These variables X_{11}, X_{12} and X_{13} are denoted respectively "above," "across," and "below" for their relationship of the source and receiver to the mixed layer depth.

By including these variables in the regression in this manner the relative effects of the various levels of sea state, of mode, and of position of the source and receiver on the mean directional difference in TL were estimated. The individual $\Delta TL_{(ra)}$ were then adjusted by the estimate of the expected difference under the appropriate conditions. The collection of adjusted differences in TL given by

$$\hat{Y}_{adj}(\Delta t) = \Delta TL_{(adj)} = \Delta TL_{(ra)} - \hat{\mu}_{\Delta} \tag{12}$$

forms the basis for the variance analysis described below.

In addition the relative effects of sea state, of mode, and of position of the source and receiver on expected directional difference in TL are interesting in their own right. The results of this analysis will be presented in section VI-b.

b. <u>Analysis of Acoustic Variability</u>.

The determination of the variation in TL as a function of selected environmental and system parameters is the principal aim of this study. This was approached using two different though complimentary techniques. The first technique was to perform a multiple regression on $(\Delta TL_{(adj)})^2$ in a manner similar to that presented in the

section on means analysis. Again each frequency was treated independently and the independent variables were the same as given in Table 1 except that time between transmissions (Δt) and $(\Delta t)^2$ were also included.

Since $E((\Delta TL_{(adj)})^2) = \sigma^2_{TL}$ the variability in TL under any specific set of conditions can be estimated by

$$\hat{\sigma}^2_{TL} = \frac{1}{n} \sum_{i=1}^{n} (\Delta TL_{(adj)})^2 \tag{13}$$

by averaging all data collected under that specific set of conditions. Similarly multiple regression of the variable $(\Delta TL_{(adj)})^2$ on a set of selected independent variables will yield estimates of the quantitative effects the various variables have on the variability in TL and the magnitude of the variability in TL can be estimated for any situation described by the explanatory or independent variables. Further elaboration on this analysis technique and the interpretation of the results are contained in Appendix II.

The second technique used was to use subsets of the entire collection of data to estimate the function $Var(Y_{adj}(\Delta t))$ for a given set of conditions where the particular data in each subset was dictated by the specific conditions to be considered. The function $Var(Y_{adj}(\Delta t))$ was calculated for any particular subset of data and hence set of conditions as a function of t over the range $0 \leq t \leq 90$ minutes by partitioning the data within the subset on the basis of nine 10-minute intervals.

For each time interval

$$\hat{\sigma}_t^2 = \frac{1}{n} \sum_{i=1}^{n} (\Delta TL_{(adj)})^2 \tag{14}$$

was calculated. On this basis the dependence of variability in TL on time was determined for a given set of conditions. By again pooling over time within a given set of conditions the relative magnitudes of variability in TL was determined for a variety of sets of conditions.

In this manner the nature and magnitude of acoustic variability was examined as a function of time of year; area in the ocean; general thermal structure type; transmission range, mode, and frequency; sea state; position of source and receiver with respect to the layer depth; and combinations thereof.

VI. Results.

A. Analysis of Acoustic Variability - Multiple Regression.

The results of two multiple regressions to determine the quantitative relationship of variability in TL to selected parameters are presented in Tables 2 and 3. Table 2 depicts the results when all available data was analyzed. Table 3 depicts the results when the data analyzed consisted of all data with the source depth in the mixed layer and no deeper than 50 feet. Hence Tables 2 and 3 describe respectively the quantitative relationships of factors contributing to acoustic variability over the entire range of conditions investigated and over a more selected range of conditions encountered by surface ship sonar. The effect of sea state on acoustic variability is more appropriately assessed in the analysis presented in Table 3.

Table 2.

Coefficients of Multiple Regression of Acoustic Variability as a Function of Selected Parameters Over Entire Range of Experimental Conditions.*

Frequency	2.2 KC	8 KC	16 KC	25 KC
Regression Parameter	← Regression Coefficients →			
Constant	14.18	11.74	17.73	25.62
Range (KY)	.11	-.01	-.14	-.84
Sea State = 1	3.70	0.00	2.04	-1.92
Sea State = 2	3.20	1.92	.34	3.71
Sea State = 3	3.70	-.62	1.08	5.77
Sea State = 4	-10.60	-1.30	-3.46	-7.57
Transmission mode = 1	2.12	-2.33	-2.58	-6.58
" = 2	5.47	6.50	4.84	5.63
" = 3	4.10	1.07	.64	-1.46
" = 4	3.11	2.35	9.15	-4.21
" = 5	-10.50	-8.24	-11.18	14.89
" = 6	-4.32	.67	-.57	-7.93
Above Layer Depth	-.96	-5.05	-6.18	-10.90
Across Layer Depth	.70	1.61	2.40	.45
Below Layer Depth	.26	3.50	3.77	10.34
Δ Depth (100 ft)	-.76	-.28	-2.00	-3.12
Δ Temperature (°F)	-.63	.57	0.00	-.42
Average Temp Gradient(°F/100 ft)	1.70	.37	1.70	1.97
Δ Time (hrs)	-.90	2.54	18.27	47.01
$(\Delta\ \text{time})^2$	.00	2.15	-2.61	-23.93

*See Appendix II for discussion of use of this table.

Table 3.

Coefficients of Multiple Regression of Acoustic Variability as a Function of Selected Parameters for Near Surface Transmission Sources.*

Frequency	2.2 KC	8 KC	16 KC	25 KC
Regression Parameter	← Regression Coefficients →			
Constant	7.03	16.96	18.39	28.49
Range (KY)	-.04	-.35	-.94	-1.14
Sea State = 1	3.50	.68	5.06	3.09
Sea State = 2	4.56	.56	-2.77	-.73
Sea State = 3	2.67	-.93	.64	6.51
Sea State = 4	-10.74	-.31	-2.93	-8.87
Transmission mode = 1	-1.82	-6.38	-9.86	-6.61
mode = 2	7.27	4.85	.48	2.64
mode = 3	2.20	-1.94	-2.98	-1.51
mode = 4	1.00	1.90	9.03	-2.86
mode = 5	-----	-----	-----	-----
mode = 6	-8.72	1.52	2.82	8.35
Above Layer Depth	.63	-2.85	-3.44	-4.57
Across Layer Depth	-.63	2.85	3.44	4.57
Below Layer Depth	-----	-----	-----	-----
Δ Depth (100 ft)	-.39	-.26	-1.87	-1.77
Δ Temperature (°F)	-.57	.69	.05	.25
Average Temp Gradient(°F/100 ft)	.94	-.08	1.37	.67
Δ Time (hrs)	32.90	1.50	24.19	19.69
$(\Delta \text{ time})^2$	-22.44	1.73	-10.03	-7.51

*See Appendix II for discussion of use of this table.

From Tables 2 and 3 the following can be inferred.

1. Multiple Regression Constants.

The constants of the multiple regressions, while representing the general trend of the magnitude of acoustic variability as a function of frequency as will be seen in later results, are not values of expected variability for the various frequencies. These constants represent the base values for the respective frequencies to which adjustments are made according to the prevailing conditions of transmissions to estimate the acoustic variability expected under those conditions. The particular adjustments to be made are a function of the values of the regression parameters and their corresponding regression coefficients.

2. Range.

Acoustic variability generally decreases with range with the rate of decrease increasing with frequency. This effect is stronger in the upper layers as seen by comparing the values of Table 3 with those of Table 2. These results suggest that while systematic or coherent variations in density structure may occur over short ranges causing substantial variation in the amount of energy focused on the receiver the accumulated effect of such charges over longer ranges tends to be less coherent. Consequently, the resulting amount of acoustic energy focused on the receiver at longer ranges is more likely to be subject to random changes than systematic changes and as a result will experience less overall variation.

The effect of increasing frequency amplifying the rate of decrease in acoustic variability with range suggests that the magnitude of acoustic variability may be a function of the relationship of wave length of the signal to the dimension of the perturbations in the density structure. The lower frequencies have longer wavelengths and would be less affected by density irregularities of smaller dimension while the higher frequencies with their correspondingly shorter wave lengths would experience a relatively greater effect. In addition, this relationship of wave length to the dimension of temporal and spacial irregularities in the density structure at least partly explains the relationship of the magnitude of overall acoustic variability to frequency.

3. Sea State.

In Table 3 acoustic variability generally decreases with increasing sea state. As sea state is measured subjectively the four levels of sea state are only considered as qualitative variables. The values in Table 3 show the relative changes in acoustic variability for the four sea states under which the experimentation was conducted. The results suggest that as sea state increases the density structure becomes more consistent in time and space and acoustic variation decreases. The data in Table 2 with respect to sea state is not representative of the true effect of sea state since data was included in that analysis which was not influenced by changes in sea state.

4. <u>Transmission Mode</u>.

The effect of transmission path type or mode on acoustic variability is given in both Tables 2 and 3. Any differences which appear between the two tables are due to a lack of deep source data in Table 3.

By considering the entire range of conditions investigated as represented in Table 2 transmission associated with modes 1, 5 and 6 is generally less variable than transmission associated with modes 2, 3 and 4. Modes 1 and 5 are direct transmission uninfluenced by surface and bottom reflection effects and relatively unaffected by changes in the refraction characteristics of the local medium unless the receiver is in the vicinity of the edge of the shadow zone. Mode 6 is a bottom bounce path and the long distances traveled on this path through the relatively stable deep ocean density structure results in little acoustic variation. Mode 2 is transmission in the thermocline and associated with the defraction of the limiting ray. Since the position of the limiting ray is greatly influenced by the action of internal waves at the top of the thermocline mode 2 transmission is highly variable. Modes 3 and 4 are associated with one or more surface bounces respectively and if transmission crosses the thermocline the effect of internal waves is also encountered. Both of these factors contribute to high variability.

By comparing the values in Tables 2 and 3 it is observed that in general transmission via modes 1, 3 and 4 are less variable if only near surface sources (<50 ft) are considered than if all conditions are considered. In addition it is observed that with the source near the surface mode 2 TL variability decreases and mode 6 TL variability increases at all frequencies except for 2.2 KC relative to the values obtained when using data under all conditions. Mode 5 did not occur when the source was within the mixed layers.

5. Source/Receiver Position.

In Table 2 it is observed that the effect of relative position of the source and receiver with respect to the layer depth is quite consistent over frequency. Acoustic variability is least within the mixed layer and highest when the source and receiver are both below the mixed layer. The magnitude of this effect increases markedly with increasing frequency. A similar effect is evident in Table 3 except that no transmissions with source and receiver below the mixed layer were included in the analysis. Similar effects are observed when comparing acoustic variability as a function of transmission mode.

6. Depth Differential, Temperature Differential and Average Temperature Gradient Between Source and Receiver.

Acoustic variability can be seen to consistently decrease with increasing depth differential between the source and receiver for all frequencies. The effect of the temperature change between the

source and receiver is mixed for the various frequencies but the average temperature gradient between the source and receiver depths has a positive effect on acoustic variability for all frequencies.

7. Time.

It was suggested in Section I that the primary source of acoustic variation is the temporal and spacial variation in the density structure of the acoustic environment directly between the source and receiver. It was also noted in Section III that for the available data each observed ΔTL contained a spacial variability component since it was impossible to obtain pairs of transmission losses with identical source and receiver positions. It was also suggested that ΔTL might contain a temporal variability component in that the magnitude of $Var(\Delta TL)$ would depend upon the length of the time interval between observations of the individual paired TL's when all other conditions were held constant. Tables 2 and 3 suggest that this is indeed so.

The relationship of acoustic variability to time is represented here as a quadratic function. No attempt was made to fit higher order polynomials. Consequently, it is not suggested that the true functional relationship between time and acoustic variability is expressed in Tables 2 and 3. However, the suggestion that the nature of this relationship depends upon frequency and also on the particular set of conditions considered is presented. Also the

evidence is that in general variability increases with time up to about 30 to 60 minutes and then levels off or maybe even decreases. These time related effects were examined further using the second variance analysis technique presented below.

B. Analysis of Acoustic Variability as a Function of Selected Parameters.

As mentioned in Section V acoustic variability was also investigated by dividing the data into subgroups on the basis of the value of various environmental and system parameters and the overall acoustic variability measured and compared for each value of the parameter considered. Any effects of time within a subgroup were also investigated by calculating observed acoustic variability in 10 minute intervals along the time scale representing time between paired observations in TL.

This approach is very versatile in that it only depends upon subdividing the data base on the basis of selected parameters. Fine subdividing, however, can lead to a sparcity of observations in any given case and hence reduce the precision of the corresponding estimate of acoustic variability.

The results of this analysis technique are generally complementary to that obtained in the multiple regression analysis but additional cases are considered. It should be pointed out that in contrast to the multiple regression analysis which was performed to determine the relative effects of the value of selected parameters on acoustic

variability the present analysis procedure estimates the observable acoustic variability for selected parameters or sets of conditions averaged over all values of all other parameters. Hence, in the present analysis two cases are not really comparable unless the distribution of conditions covered by the two data sets were similar with the exception of those parameters or sets of conditions which formed the basis of the subdivision of the data.

The results of this analysis technique is presented below.

1. Frequency.

The data base was initially subdivided on the basis of signal frequency. The distribution of conditions encountered under all frequencies was essentially the same. Hence, the estimates of the magnitude of acoustic variability as a function of frequency as presented in Table 4 are comparable. In Table 4 and all ensuing tables of the same nature the numbers in parenthesis are the sample sizes upon which the computations were based.

Table 4.

Magnitude of Acoustic Variability by Frequency Averaged Over all Experimental Conditions.

frequency	2.2 KC	8 KC	16 KC	25 KC
Acoustic variability (decibel)	15.80 (1369)	12.21 (3631)	13.41 (2394)	26.34 (2596)

One possible contributor to the observed non-monotonic relationship of acoustic variability as a function of frequency which is in contrast to the generally accepted notion that variability will increase with frequency is the difficulty in measuring low frequency transmission loss with 1949-1952 era equipment. Consequently measurement error may have been such to increase the variability in TL at 2.2 KC. However, to create the observed relationship of variability with frequency the standard deviation of measurement error would have to be on the order of 2-3 decibels higher at 2.2 KC than at the other frequencies. Thus it is felt that the observed relationship is not spurious but is caused by the nature of the ocean as an acoustic environment.

2. Time.

Figure 5 represents the observed relationship of acoustic variability as a function of time between transmissions.

The relationship expressed here is similar in form to that given by the multiple regression approach on the entire data base. There is a general increase in variability with time up until 30 to 60 minutes and then a leveling off or possible decrease. Sample size decreased considerably however for the upper time intervals reducing the precision of estimate of variability in those cases. The data and sample sizes associated with Figure 5 are presented in Table 3A of the Appendix.

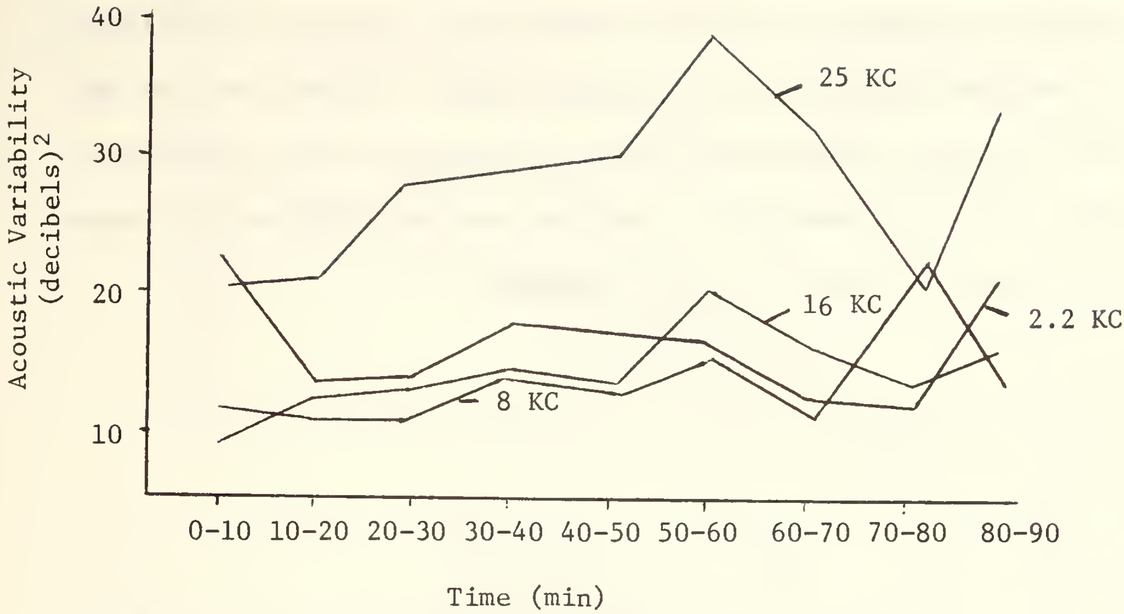

Figure 5.

Acoustic Variability by Time and Frequency

Since there was a significant difference in the nature and magnitude of acoustic variability as a function of frequency all ensuing analyses will be performed separately for each frequency.

3. Transmission Range.

The effect of transmission range on acoustic variability as a function of frequence is presented in Figure 6.

It is observed that except for frequency 25 KC acoustic variability is lower in the first 2 KY's than immediately thereafter. This is probably a result of the dominance of the relatively stable transmission along direct transmission paths (mode 1) at these short ranges. Acoustic variability at 2.2 KC fluctuates highly

as a function of range but no obvious trend appears. Conversely acoustic variability at 25 KC behaves quite regularly and displays a prominent decrease with increasing range. Frequencies 8 KC and 16 KC are more consistent than 2.2 KC but show less of a decrease with range than does 25 KC. The data upon which Figure 6 is based is presented in Table 4A of the Appendix.

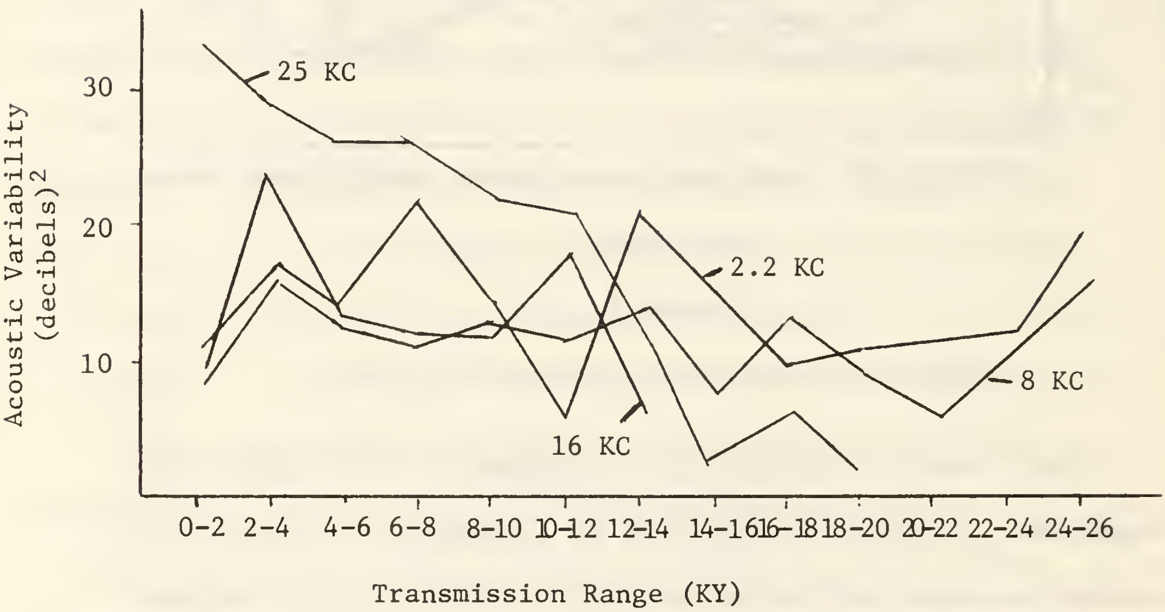

Figure 6.

Acoustic Variability by Range and Frequency.

4. <u>Sea State</u>.

The effect of sea state on variability in TL is presented in Figure 7.

It can be seen that there is a marked decrease in acoustic variability at the higher seastates similar to that observed in the

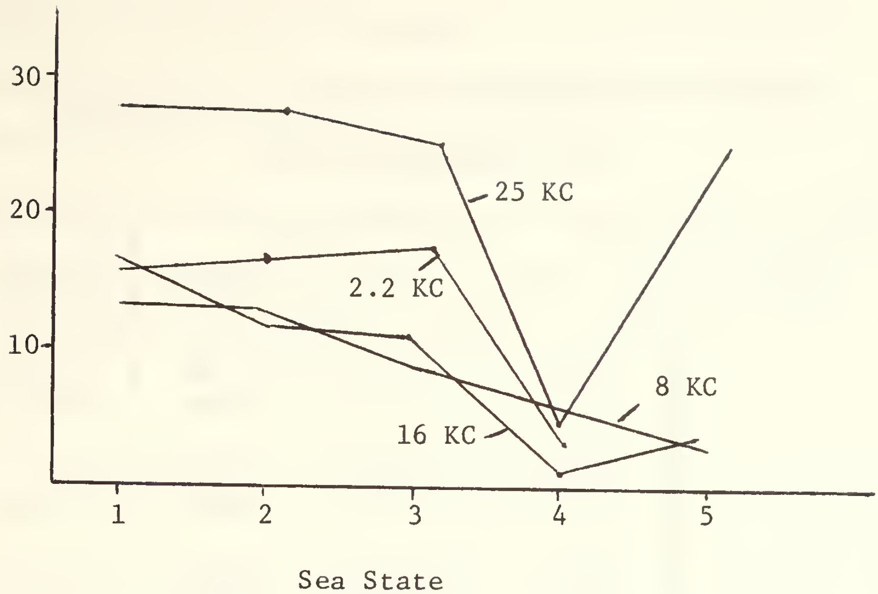

Figure 7.

Acoustic Variability by Sea State and Frequency.

multiple regression analysis. The apparent increase in variability at sea state 5 for frequency 25 KC is based on a very small sample and hence may not be representative of the true situation. The data upon which Figure 7 is based is presented in Table 5A of the Appendix.

5. Transmission Mode (Path type).

The magnitudes of acoustic variability as a function of frequency and transmission modes are presented in Table 5.

The acoustic variability by frequency averaged over mode is also presented for comparison purposes. It can be seen that the effect of frequency on acoustic variability depends upon the transmission mode. The observed relative effects are reasonably consistent over frequency.

Table 5.

Acoustic Variability as a Function of Frequency and Transmission Mode.

Transmission Mode	2.2 KC	8 KC	16 KC	25 KC
1	16.60 (293)	8.70 (1014)	9.59 (984)	21.96 (976)
2	20.50 (228)	21.08 (322)	21.99 (289)	44.36 (338)
3	18.74 (241)	11.44 (890)	13.50 (706)	25.11 (767)
4	18.68 (71)	12.26 (244)	20.96 (63)	18.34 (91)
5	4.21 (14)	5.68 (38)	4.44 (12)	47.40 (17)
6	11.87 (522)	13.66 (1123)	15.94 (340)	25.09 (409)
Averaged Over Mode	15.80 (1369)	12.21 (3631)	13.41 (2394)	26.34 (2596)

It should be noted that the various transmission modes do not themselves result in different magnitudes of acoustic variability but that the environmental conditions and system parameters which give rise to different transmission modes also gives rise to different magnitudes of variation. Thus to interpret the information in Table 5 it is necessary to remember the nature of the individual paths types designated as the 6 transmission modes.

6. <u>Source and Receiver Depths Relative to the Layer Depth</u>.

In the multiple regression analysis it was observed that the magnitude of acoustic variability depended upon the relationship of the source and receiver depths to the layer depth. A similar effect was observed in the present analysis by subdividing the data on the basis of source, receiver and layer depths. The results of this analysis are presented in Table 6.

Table 6.

Acoustic Variability as a Function of Location of Source and Receiver with Respect to the Layer Depth and of Frequency.

Source/receiver Locations	2.2 KC	8 KC	16 KC	25 KC
Above Layer Depth	17.25 (330)	8.04 (1377)	8.62 (956)	16.55 (1001)
Across Layer Depth	15.52 (382)	13.90 (1037)	15.74 (659)	28.46 (721)
Below Layer Depth	15.24 (657)	15.50 (1217)	17.39 (779)	35.77 (876)

It can be seen that variability is considerably less above the layer depth except in the case of 2.2 KC which appears relatively independent of source/receiver position. Also "above" the layer depth variability at 2.2 KC and 25 KC is double that at 8 KC and 16 KC whereas "across" and "below" the layer depth variability at 25 KC is double that at the other 3 frequencies.

7. Season.

To assess differences in acoustic variability as a function of season the data was subdivided into 4 groups corresponding to Jan.-March, Apr.-June, July-Sept. and Oct.-Dec. Little data was available in the Oct.-Dec. time period. The results of this analysis are presented in Table 7.

Table 7.

Acoustic Variability as a Function of Season and Frequency.

Season	2.2 KC	8 KC	16 KC	25 KC
Jan.-March	14.61 (301)	9.90 (1645)	11.33 (1163)	17.80 (1174)
Apr.-June	16.42 (213)	16.48 (650)	15.64 (451)	29.62 (506)
July-Sept.	16.07 (855)	12.74 (1306)	15.19 (759)	35.58 (885)
Oct.-Dec.	-----	23.71 (30)	17.05 (21)	31.53 (33)

It is observed that the general pattern of non-monotonicity of acoustic variability with frequency is maintained through the first 3 seasons and that during the Jan.-March time period variability is the least. This latter effect is probably a result of the relatively high sea states and deep mixed layers experienced during this season.

8. Region.

Since regional as well as seasonal differences exist the data was subdivided on the basis of geographical location. Five areas were selected on the basis of the defination of major water masses. The defination of these five areas is given in Figure 8. The results of the analysis by region are given in Table 8.

Figure 8.

Geographical Regions Selected for Acoustic Variability Analysis.

The three regions with the majority of the available data show the same non-monotonicity of acoustic variability with frequency.

Region 1 is the area in which the least acoustic variability is encountered and regions 4 and 5 are the most acoustically variable.

Table 8.

Acoustic Variability as a Function of Geographical Region and Frequency.

Geographical Region	2.2 KC	8 KC	16 KC	25 KC
1	14.55 (1033)	10.51 (2060)	12.26 (1371)	25.58 (1454)
2	-----	25.05 (82)	18.63 (53)	40.34 (56)
3	-----	13.11 (249)	17.56 (180)	24.53 (173)
4	19.21 (164)	14.99 (693)	14.09 (426)	24.33 (514)
5	20.09 (172)	12.78 (547)	14.18 (364)	30.47 (401)

9. Region by Season.

The regional and seasonal effects were further assessed by comparing seasonal affects within area. These results are presented in Table 6A of the Appendix. Empty cells in the table are due to the lack of available data in that season-region combination. In general the winter is the least acoustically variable time in all regions and this effect is most prominent and consistent in region 1.

C. Summary of Factors Affecting Acoustic Variability.

The observed relationships of acoustic variability as a function of selected parameters are summerized as follows.

1. Frequency.

Acoustic variability is non-monotonic with frequency. The ranking of frequency with increasing acoustic variability is 8 KC, 2.2 KC, 16 KC and 25 KC. In general any other parameters affecting acoustic variability has relatively less effect on 2.2 KC transmission and relatively more effect on 25 KC transmission. In most cases investigated the variability of 2.2 KC transmission did not behave in a manner compatible with that of the other frequencies in either magnitude or in response to varying the value of selected parameters.

2. Range.

Acoustic variability decreases with transmission range and this effect is most prominent in the higher frequencies.

3. Sea State.

Acoustic variability decreases with increasing sea state with a marked decrease at sea states ≥ 4.

4. Transmission Mode (Path type).

Acoustic variability varies as a function of the type of transmission path which occurs. Transmission along the various path types encounter different physical phenomena giving rise to variation in direction and magnitude of acoustic energy transmission.

It is observed that transmission which encounters neither the surface or bottom (modes 1 and 5) is the least variable. Transmission by defraction of the limiting ray (mode 2) is the most variable. Contact with the surface (modes 3 and 4) increases observed variability and transmission through the relatively stable deep waters on a bottom bounce path (mode 6) yields relatively stable transmission.

5. Source and Receiver Position with Respect to Layer Depth.

Acoustic variability when both the source and receiver are in the mixed layer is the least variable situation while transmission with both the source and receiver below the mixed layer is the most variable situation.

6. Depth Differential Between Source and Receiver.

Acoustic variability decreases with increasing depth differential between source and receiver.

7. Average Temperature Gradient.

Acoustic variability increases with increasing average temperature gradient measured from the source and receiver depths.

8. Time.

Acoustic variability in general increases with time up to a point in the neighborhood of 30-60 minutes depending upon frequency. In certain cases there is also a tendency for acoustic variability to decrease again at higher values of time but this observed effect may be spurious in light of the small sample sizes at high values of time.

9. Season.

Acoustic variability is least in the winter months and approximately the same in spring and summer. Insufficient data was available to measure variability in the fall.

10. Region.

Acoustic variability is least north of 45°N in the Atlantic and highest on the mid-North Atlantic regions. The seasonal trends occur in all regions with winter in the far-North Atlantic experiencing the least variability of any season-region combination.

D. Summary of Physical Phenomena Inferred to Influence the Magnitude of Acoustic Variability.

On the basis of the observed nature of acoustic variability as a function of selected parameters several causal factors may be inferred.

To account for the differences in acoustic variability as a function of frequency it is postulated that the degree of variability experienced in any given situation depends upon the relationship of the wave length of the acoustic energy to the dimension of density perturbations occurring under that situation.

To explain the decrease in acoustic variability with increasing range the nature of the effect of density structure perturbations on acoustic transmissions must be considered. Even though the main density structure features are assumed relatively constant over the times and ranges considered, small scale perturbations on this density structure

exist. These perturbations affect the refraction of the acoustic energy at the point of the perturbation and the integration over distance of all such effects contributes to the variation in acoustic energy ultimately reaching the receiver. It is inferred from the observed decrease of variability with range that this integrated effect will typically be smaller at longer ranges due to the nullifying effect of numerous independent perturbations than the similar effect over shorter ranges where such a nullification process would not be so prevalent. The suggestion that decreased variability with increasing range is a function of the grazing angle of the energy path with the surface [1,7] is not supported by the present study. The observed effect occurs for all path types including those which do not contact the surface. This does not preclude the grazing angle from being related to acoustic variability but it does suggest that it is not the only source of such variation.

To account for the decrease in acoustic variability with increasing sea state it can be inferred that at low sea states the density structure is relatively stable. Consequently, any perturbations, whether they are systematic as in the action of internal or surface waves or just random turbulence, will cause relatively greater acoustic variation than if the same perturbations were superimposed or are already agitated and homogeneous acoustic environment created by a high sea state.

To account for the nature of acoustic variability as a function of the relationship of source and receiver depths to the layer depths

it was noted that when transmitting entirely in the mixed layer the acoustic energy is traveling through a region which is reasonably well mixed due to wave action. Thus the total energy received at the receiver is already a function of the accumulated effect of many random perturbations and further perturbations will not have a significant effect. Conversely, when transmitting below the mixed layer in the more structured water of the thermocline any perturbations, particularly those associated with internal waves, can have a considerable effect on the amount of energy which ultimately reach the receiver. When transmitting from the mixed layer into the thermocline at least part of the transmission is within the mixed layer which leads to a degree of acoustic variability intermediate between the cases when both the source and receiver are above or both are below the layer depth.

The relative magnitudes of acoustic variability as a function of transmission mode can be explained on the basis of the nature and magnitude of the various sources of the density perturbations encountered during transmission. The direct transmissions of modes 1 and 5 are the least affected by such perturbations. If direct transmissions occurs across the layer depth then the increased acoustic variability observed is due at least in part to the action of internal waves. Interaction of a transmission with the surface also increases the observed variability.

The effect of time on acoustic variability is also interesting. The observed change in acoustic variability as a function of time

between paired observations suggests that there is a significant temporal component to variability superimposed upon the spacial variability. Typically this temporal variability increases up to 30-60 minutes ofter which it levels off or in some cases appears to decrease. The increase up to 30-60 minutes suggests that the range of density changes that occur in this time period are of the same magnitude as those which occur spacially in a given acoustically homogeneous region at a fixed time. Any decrease in acoustic variability at a later time suggests some cyclic phenomenon such as internal waves is bringing the acoustic environment back to an earlier state. Such decreases in variability with time, if they exist, are masked by the spacial component of variability inherent in the data. Another experiment would have to be designed to investigate these time phenomena further.

The observed dependence of acoustic variability with season and region is a result of the dominant environmental conditions for the time and place in question. The deep isothermal surface layers in the North Atlantic in the winter accompanied by high sea states are conditions which lead to consistent transmission conditions. The mid-North Atlantic in the spring and summer months develop well established thermoclines with their associated internal waves and random perturbations on the fairly stable density structures lead to a much higher degree of acoustic variability.

E. Analysis of Mean Directional Difference Multiple Regression.

As mentioned in Section III transmission loss downward from point A to point B may not necessarily equal transmission loss upward from point B to point A. However, in order to measure acoustic variability as a function of differences in transmission losses the expected transmission loss in both cases must be the same. Since each pair of observations in the analysis consisted of a "downward" minus an "upward" transmission a correction was required so that the expected difference in TL was zero.

This correction was accomplished by estimating the mean directioned difference in TL as a function of selected parameters using multiple regression techniques. Then prior to performing the variance analyses discussed above for each paired set of observations of TL the observed value of "downward" TL minus "upward" TL was corrected by the estimated mean directional difference in TL for the appropriate set of conditions. The analysis was performed using the entire data base and the results of this analysis are presented in Table 9.

The only parameters which have a consistent effect over frequency are sea state and the location of the source and receiver with respect to the layer depth.

It is seen that in general TL "downward" exceeds TL "upward" for low sea states and that the opposite effect is observed for high sea states. It is also observed that TL "downward" exceeds TL

Table 9.

Coefficients of Multiple Regression of Mean Directional Difference in Transmission Loss as a Function of Selected Parameters.

Frequency	2.2 KC	8 KC	16 KC	25 KC
Regression Parameters	←Regression coefficients→			
Constant	-.26	-.19	.64	-.04
Range (KY)	.03	.01	-.01	.01
Sea State = 1	.32	.10	.46	.35
" = 1	-.10	.02	-.09	.01
" = 3	-.11	-.05	-.23	.07
" = 4	-.11	-.07	-.14	-.44
Transmission Mode = 1	.06	.17	-.33	.25
" = 2	.25	.46	-.18	.79
" = 3	.17	.03	-.39	.05
" = 4	-.41	-.07	-.12	.21
" = 5	.00	-.84	.69	-1.37
" = 6	-.08	.25	.34	.06
Above Layer Depth	.22	.10	.10	.21
Across Layer Depth	0	-.02	.17	-.03
Below Layer Depth	-.22	-.08	-.27	-.18
Δ Depth (100 ft)	.27	-.08	-.08	.01
Δ Temp (°F)	.17	.05	.02	.03
Average Temp Gradient (°F/100 ft)	-.08	.01	-.03	-.05

"upward" when the source and receiver are in the mixed layer with the opposite effect being observed when both the source and receiver are below the mixed layer.

Both these effects appear to be a consequence of the nature and magnitude of the turbulence in the acoustic medium at the depths of the source and receiver and the resulting effect on the dispersion of acoustic energy.

F. Further Analysis.

1. Large TL Analysis

In the multiple regression analysis of acoustic variability a significant amount of the total dispersion of the dependent variable $(\Delta TL)^2$ was accounted for by the regression. Even so a sizable residual error remained after fitting the model. This large residual error would appear to be inherent to any approach to the problem based solely on statistical properties of the changing acoustic environment as opposed to the actual changes experienced. However, it is not known how to measure these actual changes nor how to use this information if it were available. Hence, a statistical approach seems to be required.

In such a statistical approach the only way to reduce the residual error is to construct a better model for the relationship of acoustic variability to the acoustic environment. In a search for additional factors which could cause or contribute to the larger values of ΔTL (say >10 DB) which made up approximately 10% of the data base, the following analysis was performed.

The majority of the large changes in TL are suspected to be a result of the action of internal waves in the thermocline as this is the only short time scale force which make appreciable changes in the environmental structure. One possible phenomena which might occur under the action of internal waves is a shift in the shadow zone. If the source or receiver were located in the vicinity of the edge of the shadow zone considerable variation in TL could be experienced with a shift in the shadow zone. With this in mind, in all cases when a shadow zone occured, its location was predicted on the basis of local temperature profile data. For each observed TL in these cases the ratio of the transmission range to the range from the source to the edge of the shadow zone at the depth of the receiver was determined. The data was then subdivided on the basis of this ratio.

It appears that for ratios < 1 (i.e. the transmission range is less than the shadow zone range) the variability was greater than that when the ratio was in the interval 1 to 2 which is the range occupied by the shadow zone. This leads to the inference that the movement of the shadow zone is not a significant factor in causing acoustic variability. On the other hand the magnitude of the variability observed short of the shadow zone suggests sizable spacial and/or temporal density structure perturbations in this region.

2. <u>The Consequence of Using a Linear Adjustment for the Range Differential</u>.

As mentioned in Section III both paired observations of TL did not necessarily have exactly the same transmission range and that to account for this a linear correction was used. To determine if this procedure contributed to the variability of ΔTL the magnitude of acoustic variability was calculated as a function of the difference in range between the paired transmissions.

The result of this analysis gave no evidence that the linear correction procedure contributed to acoustic variability.

3. <u>The Distribution of ΔTL</u>.

The histogram of ΔTL considering data observed over the entire range of conditions is bell shaped and symmetrical. However, since the variability in TL depends upon the situation considered this histogram is displaying data from a wide range of distributions with different variances. As a consequence while it is reasonable to assume that TL is normally distributed such inferences cannot be made solely on the basis of this data.

To make such inferences relative large sample sizes of TL would be required under each environmental situation of concern.

VII. Conclusion.

Two important topics were considered in this study. The first dealt with determining the nature and magnitude of acoustic variability in the ocean as a function of selected environmental and system

parameters. The second topic dealt with inferring the nature of the physical processes inherent in the ocean which conceivably caused the observed variation.

It has been inferred that the nature and magnitude of the changes in density structure in the ocean as a result of surface induced turbulence, internal waves and other structural perturbations inherent to a dynamic body of water have caused the observed acoustic variability. The specific parameters which were observed to have influenced this acoustic variability are transmission frequency, transmission range, sea state, transmission path, the location of the source and receiver with respect to the layer depth, the depth differential and average temperature gradient between the source and receiver and the time between transmissions.

The nature of the available data presented many difficulties in the analysis and limited the range of inferences which could be drawn, especially with respect to the effect of time on variability. However, the author feels that the consistency of the nature of the observed variability with the available knowledge of the nature of the physical processes in the ocean and the magnitude of the sample size suggests that the observed nature of acoustical variability and the inferred causes thereof are authentic.

APPENDIX I

Table 1A

Data Recorded During AMOS Experiment.

1. Date of observation.
2. Time of observation.
3. Range of observation.
4. Source depth.
5. Receiver depth.
6. Propagation loss at 2.2 KC, 8 KC, 16 KC, 25 KC.
7. Sea state at source ship.
8. Sea state at receiving ship.
9. Water depth.
10. Water temperature, every 10 feet to 100 feet and every 50 feet to 450 feet. (Temperature profiles were observed from both ships at least once an hour conditions permitting.)

Table 2A

Data Accuracies

Variable	Accuracy
time	- to nearest minute
range	- to nearest five yards
source and receiver depth	- to nearest foot
station location	- to nearest minute in Latitude and Longitude
water temperature	- to nearest 1/10th of a degree
transmission loss	- to nearest decibel

Table 3A

Acoustic Variability as a Function of Time and Frequency

Time Interval	2.2 KC	8 KC	16 KC	25 KC
0-10 min	22.81 (71)	11.62 (419)	9.43 (204)	20.75 (227)
10-20	13.92 (412)	11.12 (1202)	12.50 (797)	21.22 (866)
20-30	14.56 (271)	11.03 (758)	13.21 (478)	28.10 (536)
30-40	17.89 (233)	14.12 (550)	14.28 (373)	29.13 (404)
40-50	17.31 (130)	12.73 (315)	13.30 (226)	20.47 (242)
50-60	16.62 (118)	15.15 (198)	20.70 (152)	39.17 (165)
60-70	12.98 (63)	11.73 (101)	15.84 (82)	33.02 (86)
70-80	12.96 (43)	23.44 (60)	13.19 (52)	21.01 (49)
80-90	21.96 (20)	13.50 (28)	16.03 (25)	34.11 (23)

Table 4A

Acoustic Variability as a Function of Transmission

Range and Frequency.

Transmission Range	2.2 KC	8 KC	16 KC	25 KC
0-2 KY	9.65 (189)	8.58 (402)	10.81 (431)	32.37 (399)
2-4	24.10 (303)	15.01 (747)	16.27 (730)	29.27 (753)
4-6	14.01 (354)	12.35 (738)	13.51 (654)	25.36 (706)
6-8	21.02 (101)	10.31 (271)	11.42 (206)	25.54 (259)
8-10	14.07 (142)	12.54 (458)	12.44 (251)	21.27 (305)
10-12	5.12 (7)	11.71 (107)	17.56 (49)	20.43 (67)
12-14	20.63 (7)	14.37 (173)	6.66 (91)	10.58 (98)
14-16	-----	7.10 (55)	-----	3.05 (5)
16-18	9.53 (59)	12.05 (166)	-----	6.37 (5)
18-20	10.15 (101)	9.87 (281)	-----	2.69 (1)
20-22	-----	5.61 (16)	-----	-----
22-24	12.71 (59)	11.94 (86)	-----	-----
24-36	19.40 (47)	16.14 (113)	-----	-----
26-28	-----	10.75 (4)	-----	-----
28-30	-----	2.53 (1)	-----	-----
30-32	-----	11.23 (13)	-----	-----

Table 5A

Acoustic Variability as a Function of Sea State and Frequency.

Sea State	2.2 KC	8 KC	16 KC	25 KC
1	15.76 (688)	13.12 (1534)	16.09 (1011)	27.31 (1119)
2	15.95 (459)	13.19 (1417)	12.51 (154)	27.77 (1014)
3	17.28 (196)	8.94 (539)	11.04 (339)	25.05 (359)
4	3.31 (26)	5.51 (116)	1.81 (80)	4.77 (96)
5	-----	3.22 (25)	3.47 (10)	25.44 (10)

Table 6A

Acoustic Variability as a Function of Region

Within Season and Frequency.

Season	Region	2.2 KC	8 KC	16 KC	25 KC
Jan. to March	1	12.91 (129)	5.51 (690)	5.51 (519)	10.40 (497)
	2	-----	-----	-----	-----
	3	-----	13.47 (240)	17.40 (171)	26.42 (155)
	4	10.65 (81)	13.77 (393)	15.64 (268)	18.99 (302)
	5	20.53 (91)	11.93 (322)	15.34 (205)	26.82 (220)
April to June	1	19.19 (150)	18.45 (305)	17.87 (220)	39.36 (245)
	2	-----	25.83 (52)	19.66 (32)	29.71 (32)
	3	-----	3.53 (9)	20.55 (9)	11.60 (9)
	4	-----	17.08 (87)	13.49 (55)	19.68 (66)
	5	9.82 (63)	11.27 (197)	11.59 (135)	33.33 (154)
July to Sept.	1	13.90 (754)	11.47 (1065)	15.83 (632)	34.87 (712)
	2	-----	-----	-----	-----
	3	-----	-----	-----	-----
	4	27.57 (83)	16.39 (213)	10.39 (103)	37.49 (146)
	5	53.81 (18)	33.24 (28)	18.84 (24)	43.95 (27)
Oct. to Dec.	1	-----	-----	-----	-----
	2	-----	23.71 (30)	17.05 (21)	41.17 (24)
	3	-----	-----	-----	5.83 (9)
	4	-----	-----	-----	-----
	5	-----	-----	-----	-----

APPENDIX II

To estimate the effect selected system and environmental parameters have on the magnitude of acoustic variability the following multiple regression analysis was performed. The dependent variable in the regression was $(\Delta TL_{adj})^2$. This quantity was modeled as a linear function of selected explanatory parameters $X_1, X_2, \ldots, X_n$ in the form

$$(\Delta TL_{adj})^2 = b_0 + b_1X_1 + b_2X_2 + \ldots + b_nX_n + \epsilon$$

where ϵ is an error term with mean zero. In the model the values of $X_1 \ldots X_n$ are the values of the selected parameters described in Table 1 corresponding to particular values of $(\Delta TL_{adj})^2$. The multiple regression produces estimates $\hat{b}_0, \hat{b}_1, \ldots, \hat{b}_n$ of $b_0, b_1, \ldots, b_n$ such that the residual sum of squares of deviation of the data from the fitted model is minimized. The estimate

$$\hat{\sigma}^2 = \hat{b}_0 + \hat{b}_1X_1 + \hat{b}_2X_2 + \ldots + \hat{b}_nX_n$$

is then an estimate of the mean value of $(\Delta TL_{adj})^2$ at that particular set of conditions specified by $X_1 \ldots X_n$. As the mean value of $(\Delta TL_{adj})^2$ is the acoustic variability under this set of conditions, equation (1) is then an estimator of this acoustic variability.

The coefficients $\hat{b}_0 \ldots \hat{b}_n$ are given in Tables 2 and 3 for the multiple regression analyses performed. The estimate of acoustic

variability for any set of values of the explanatory parameters $X_1 \ldots X_n$ is obtained by substituting those values and the appropriate estimates of the regression parameters into equation (1). For example to estimate the acoustic variability when transmitting at a frequency of 16 KC and the following conditions:

Range - 8 KY

Sea state - 2

Transmission mode - 3

Transmission across the layer depth

Δ Depth - 200 ft

Δ temperature - 5° F

Average temperature gradient - 2.5° F/100 ft

Time between transmission - .5 hour

$$\hat{\sigma}^2 = 17.73 - (.14)(8) + (.34)(1) + (.64)(1) + (2.40)(1) - (2.00)(2)$$

$$+ (0.0)(5) + (1.75)(.25) + (18.27)(.5) - (2.61)(.25) = 28.85$$

If one compares this estimate of acoustic variability with the average acoustic variability of $13.41\ db^2$ (page 33) observed at 16 KC it is seen that this set of conditions leads to a more variable situation than experienced on the average. It should be pointed out that this overall experimental average in acoustic variability of $13.41\ db^2$ is the average over all situations which were observed but this did not represent a random sample of the variability experienced in the overall ocean.

The variability of such an estimate of acoustic variability depends upon the variances and covariances of the estimates of the $\hat{b}_i$'s in the form.

$$\mathrm{Var}(\hat{\sigma}^2) = \sum_{L=1}^{n} X_i^2 \, \mathrm{Var}(\hat{b}_i) + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} X_i X_j \, \mathrm{cov}(\hat{b}_i, \hat{b}_j)$$

The value of the variances and covariances of the $\hat{b}_i$'s is dependent upon the inherent variability of ΔTL_{adj} which is the subject of this study. Thus, these variances and covariances are not at present obtainable and as a consequence no valid estimate of the variability of the estimate $\hat{\sigma}^2$ is available.

Some sense of the validity of the model for predicting acoustic variability can be obtained by comparing the relative effects within frequency of sea state, transmission mode, and position of the source and receiver with respect to the layer depth from Table 2 with similar effects noted in Figure 7 and Tables 5 and 6. The effects across frequency of range and time from Table 2 can also be compared with the similar effects in Figures 5 and 6.

Additional support for the validity of the model is found in noting that in all cases of the multiple regression analysis there was a significant reduction in residual sums of squares due to fitting the regression at the .001 level of significance. Thus the parameters included in the model were useful in explaning the sources of the variability and hence can be inferred to be of use in predicting this variability.

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13. ABSTRACT

The temporal and spacial variation in one-way transmission loss as experienced in the ocean due to short term temporal and small scale spacial variation in the acoustic environment is examined. This variation is characterized as a function of the transmission frequency, transmission range, source and receiver depths, predominant thermal structure and geographical locality. The results obtained clearly indicate that variability in transmission loss is indeed dependent upon the relative position of source and receiver within the acoustic medium as well as the nature of the acoustic medium. Suggestions are made as to the nature of the influences which control this variation.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Acoustic						
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